

Scattering of Strings from D-branes

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We review a number of perturbative calculations describing the interactions of D-branes with massless elementary string states. The form factors for the scattering of closed strings off D-branes are closely related to the Veneziano amplitude. They show that, in interactions with strings, D-branes acquire many of their physical features: the effective size of D-branes is of order the string scale and expands with the energy of the probe, while the fixed angle scattering amplitudes fall off exponentially. We also calculate the leading process responsible for the absorption of closed strings: the amplitude for a closed string to turn into a pair of open strings attached to the D-brane. The inverse of this process describes the Hawking radiation by an excited D-brane.

Recent developments in the study of string theory at a non-perturbative level have shown that string theory is not just a theory of strings, but it also contains extended object of higher dimensionality known as the p -branes [1–9]. Since they are exchanged with the elementary strings under non-perturbative duality symmetries, the p -brane degrees of freedom are just as indispensable for the overall consistency of the theory. The p -branes have been known for some time as soliton solutions to the low-energy effective action of string theory [10–14]. In this description it was difficult, however, to investigate their ‘stringy’ properties. The situation has changed dramatically following Polchinski’s recent observation[15] that the p -branes carrying Ramond-Ramond (R-R) charges admit a remarkably simple world sheet description in terms of open strings with Dirichlet boundary conditions [16–20]. The essential idea is the following. Even in type II theories one introduces world sheets with boundaries, imposing the Neumann boundary conditions on coordinates X^m for $0 \leq m \leq p$ and the Dirichlet boundary conditions on coordinates X^M for $p+1 \leq M \leq 9$.¹ Thus, the end-points of the

open strings are free to move along a $p+1$ -dimensional hypersurface defined by $X^M = a^M$ for $p+1 \leq M \leq 9$. This hypersurface is to be thought of as the world volume of a p -brane, which has been named a Dirichlet brane (or D-brane). Since the Dirichlet boundary conditions preserve conformal invariance, the D-branes are exact solutions of the string tree level equations of motion. Therefore, this simple world sheet formulation includes the physics of all the string modes, i.e. the D-branes are exact embeddings of the R-R charged low-energy soliton solutions into string theory.

The D-branes are dynamical objects, whose transverse positions are specified by collective coordinates a^M , and whose fluctuations are described by the excitations of the open strings attached to them. Their masses scale as $1/g$ which suggests that they are responsible for the non-perturbative effects of strength $e^{1/g}$ generally present in string physics [21,22]. The fact that the D-branes carry R-R charges implies that they are exchanged with the elementary strings under duality transformations which exchange the R-R gauge fields with the NS-NS gauge fields. Indeed, many non-perturbative phenomena find a simple explanation in the language of D-branes. Interested readers are referred to [23–25] for reviews and lists of references.

The D-branes have also found an important ap-

¹It will be necessary to distinguish between components of space-time vectors that are parallel or transverse to the p -brane. In these notes we will use three different types of indices, (m, M, μ) , and follow the convention that $0 \leq m \leq p$, $p+1 \leq M \leq 9$, and $0 \leq \mu \leq 9$.

plication to the physics of black holes. Indeed, a variety of black hole solutions to low energy supergravity have an exact stringy description in terms of intersecting D-branes. Their excitations are the open strings attached to the D-branes, which allows for the counting of the black hole entropy [26–35] Furthermore, these methods give a new insight into the Hawking radiation rate of near-extremal black holes [27,36–40].

One of the main advantages of D-branes is that their dynamics admits a perturbative description in the weak coupling limit. The leading perturbative amplitudes can be computed by evaluating correlation functions on world sheets with one hole (a disk) and two holes (an annulus). Amplitudes of this kind have been computed by several authors [41–50]. In these notes, we review the simplest calculations relevant to the perturbative dynamics of D-branes and closed strings: those carried out on a disk. These amplitudes capture the physics of a single D-brane interacting with closed strings. A useful analogy to keep in mind is that of the fixed target scattering experiments designed to probe the structure of the atoms or nuclei. In this spirit, we learn something about the structure of D-branes by performing elastic and inelastic scattering experiments using a beam of closed strings.

To be more specific, we consider type II theories in a background of a single D-brane and calculate the amplitudes illustrated in figure 1. The first process corresponds to an elastic scattering of a closed string off of a D-brane. One can infer the size of these D-branes from the measurement of the scattering form factors. The second process corresponds to a change in the internal states of the D-branes. The third diagram describes an inelastic process where a closed string is absorbed by a D-brane, exciting its internal state by creating a pair of open strings. The reverse process corresponds to spontaneous emission by excited D-branes. In light of the recent realization that D-branes can be used to describe certain types of black holes [26–35], these processes model the classical absorption and Hawking radiation [27,36–40]. It will turn out that, at the formal level of evaluating correlation functions, these three types of amplitudes are closely

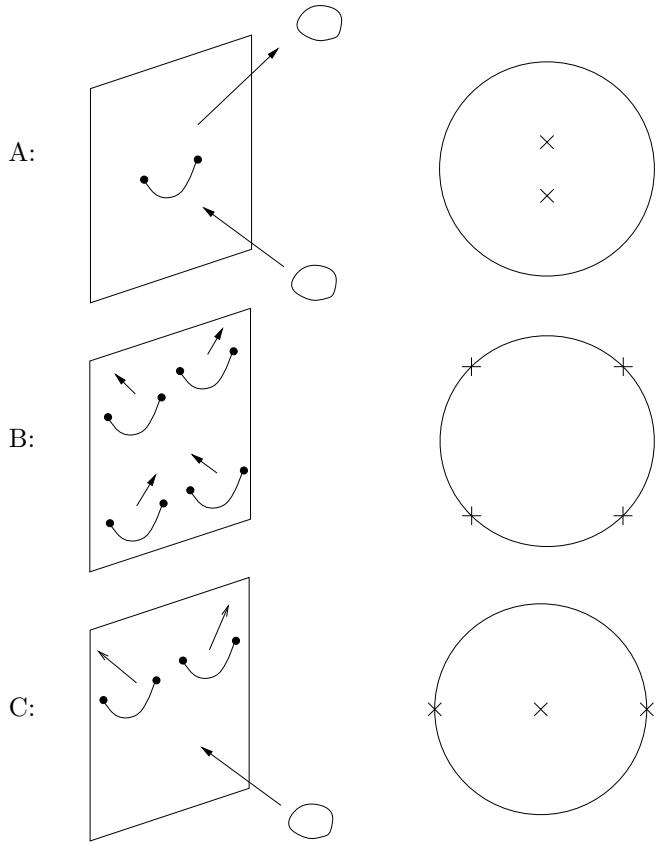


Figure 1. Schematic illustration and world sheet diagrams of the scattering processes involving a single D-brane and several string excitations, to leading order in g .

related. Once one is computed, the result can be easily adapted to find the other two.

The characteristic length scale of the D-brane form factors that we will find from our perturbative analysis is the string scale $\sqrt{\alpha'}$.² This is hardly a surprise given the manifestly stringy formulation of these amplitudes. The effective thickness of order $\sqrt{\alpha'}$ may be visualized as a ‘halo’ of open strings attached to the D-branes. The main conclusion is that the effective size of the D-branes, as seen by the elementary strings, is

²The D-instanton is a special case exhibiting a point-like structure.

a quantity of order string length which increases with the energy of the probe, thus exhibiting the Regge behavior [51].

One could ask a somewhat different question: what is the effective size of D-branes as measured by other D-branes? This question was studied in [49,50,52–56] and length-scales shorter than the string length [57] were found in cases where the coupling is weak, and the heavy D-branes move very slowly. For example, the scattering of two zero-branes reveals a non-perturbative length scale $\sim g^{1/3}\sqrt{\alpha'}$, which is nothing but the Planck length in M-theory. Thus, using D-branes as probes reveals new aspects of their substructure, but the necessary methods are non-perturbative and will not be covered here.

The organization of these notes is as follows. In section 1 we consider the scattering of massless NS-NS bosons illustrated in figure 1.A. The Regge behavior at string scale will emerge from this calculation. In section 2, we describe the extension to massless R-R bosons. In section 3, we compare the scattering amplitudes computed in sections 1 and 2 with the scattering off solitons solutions in low energy supergravity theory. The Dirichlet p -branes and the p -brane solutions give rise to identical scattering at large impact parameter, confirming that the Dirichlet p -branes are indeed the stringy realizations of the p -branes of the low-energy supergravity. In section 4, we describe how the type I four-point amplitude is related to the processes illustrated in 1.A and 1.B. Finally, in section 5, we describe how the relation between 1.A and 1.B can be extended to compute the amplitudes of type 1.C with ease. We conclude in section 6. Most of the material reviewed in these notes has appeared in earlier papers [41–44].

1. Elastic scattering of massless NS-NS bosons from D-branes

In this section, we review the elastic scattering of a massless NS-NS boson off a single D-brane [41]. We will be using the modern covariant formulation defined in terms of conformal field theories on the world sheet [58]. Nice reviews of conformal field theory approach to string theory can be found in [59–61].

Consider a type II theory in flat ten-dimensional spacetime and imagine a p -brane spanning the $X_1 \times X_2 \times \dots \times X_p$ plane. The presence of the p -brane breaks the $SO(1, 9)$ Lorentz symmetry to $SO(1, p) \times SO(9-p)$. The p -brane's mass scales as $1/g$. Therefore, to leading order in g , it is infinitely heavy and does not recoil: it is capable of absorbing an arbitrary amount of momentum in the $X_{p+1} \dots X_9$ directions without a change in its energy.

Now imagine an incoming and outgoing massless NS-NS bosons with momenta p_1 and p_2 , and polarizations ε_1 and ε_2 respectively. As we saw above, only the components of momentum parallel to the brane are conserved. Let us define two kinematic quantities invariant under $SO(1, p) \times SO(9-p)$,

$$\begin{aligned} s &= 2p_1^2 \parallel = 2p_2^2 \parallel \\ t &= p_1 \cdot p_2 \parallel, \end{aligned}$$

and compute the amplitude for a massless NS-NS state with momentum p_1 to scatter into a massless NS-NS state with momentum p_2 .

In the modern covariant formulation, scattering amplitudes are computed by evaluating the correlation function of vertex operators corresponding to the asymptotic states in the scattering process. For the process of the type illustrated in figure 1.A, the correlation function takes the form

$$A = \int \frac{d^2 z_1 d^2 z_2}{V_{CKG}} \langle V_1(z_1, \bar{z}_1) V_2(z_2, \bar{z}_2) \rangle. \quad (1)$$

The factor of V_{CKG} accounts for the volume of the conformal Killing group which is a residual gauge symmetry that survives after choosing the conformal gauge.

The form of the vertex operators is constrained by the superconformal invariance of the correlation function. For massless NS-NS states, they are given by

$$\begin{aligned} V_1(z_1, \bar{z}_1) &= \varepsilon_{1\mu\nu} :V_{-1}^\mu(p_1, z_1): : \bar{V}_{-1}^\nu(p_1, \bar{z}_1): \\ V_2(z_2, \bar{z}_2) &= \varepsilon_{2\mu\nu} :V_0^\mu(p_2, z_2): : \bar{V}_0^\nu(p_2, \bar{z}_2):, \end{aligned}$$

with

$$\begin{aligned}
V_{-1}^\mu(p_1, z_1) &= e^{-\phi(z_1)} \psi^\mu(z_1) e^{ip_1 \cdot X(z_1)} \\
V_0^\mu(p_2, z_2) &= (\partial X^\mu(z_2) + ip_2 \cdot \psi(z_2) \psi^\mu(z_2)) e^{ip_2 \cdot X(z_2)}.
\end{aligned}$$

The subscripts $\{0, -1\}$ denote the superghost charge carried by the vertex operators. The total amount of superghost charge on a disk is required to be -2 [58,62]. This requirement is a consequence of the superdiffeomorphism invariance on the string world sheet.

Now that we have all the basic ingredients laid out, all that remains to be done is to evaluate the correlation function (1), keeping in mind that Neumann boundary conditions are imposed on the directions parallel to the brane, and Dirichlet boundary conditions on the directions transverse to the brane. These correlation functions are simple (but quite tedious in practice) to compute because all the fields on the world sheet are free.

Although the original calculations of these amplitudes were performed on a disk [41], it turns out to be simpler to conformally map the disk onto a half plane where both the boundary conditions and the Greens functions take on a simple form. Let us take our world sheet to be the upper half plane \mathcal{H}^+ . The boundary conditions imposed on the real axis are

$$\begin{aligned}
X(z) &= \bar{X}(\bar{z}) \\
\psi(z) &= \bar{\psi}(\bar{z})
\end{aligned}$$

for the Neumann case and

$$\begin{aligned}
X(z) &= -\bar{X}(\bar{z}) \\
\psi(z) &= -\bar{\psi}(\bar{z})
\end{aligned}$$

for the Dirichlet case. These boundary conditions mix the holomorphic and the antiholomorphic components of the world sheet fields. Their correlation functions are given by

$$\begin{aligned}
\langle X(z)X(w) \rangle &= \ln(z-w) \\
\langle X(z)\bar{X}(\bar{w}) \rangle &= \ln(z-\bar{w}) \\
\langle \psi(z)\psi(w) \rangle &= 1/(z-w) \\
\langle \psi(z)\bar{\psi}(\bar{w}) \rangle &= 1/(z-\bar{w})
\end{aligned} \tag{2}$$

for a Neumann boundary and

$$\begin{aligned}
\langle X(z)X(w) \rangle &= \ln(z-w) \\
\langle X(z)\bar{X}(\bar{w}) \rangle &= -\ln(z-\bar{w}) \\
\langle \psi(z)\psi(w) \rangle &= 1/(z-w) \\
\langle \psi(z)\bar{\psi}(\bar{w}) \rangle &= -1/(z-\bar{w})
\end{aligned} \tag{3}$$

for a Dirichlet boundary. Now we introduce a notational device, often referred to in the literature as the “doubling trick.” The fields $X(z)$ and $\psi(z)$ are originally defined only on the half plain \mathcal{H}^+ . Let us imagine extending the definition of these fields to the full plane using $\bar{X}(\bar{z})$ and $\bar{\psi}(\bar{z})$ in a following way:

$$\begin{aligned}
X(z) &= \begin{cases} X(z) & z \in \mathcal{H}^+ \\ \pm \bar{X}(z) & z \in \mathcal{H}^- \end{cases} \\
\psi(z) &= \begin{cases} \psi(z) & z \in \mathcal{H}^+ \\ \pm \bar{\psi}(z) & z \in \mathcal{H}^- \end{cases}
\end{aligned}$$

The choice of sign depends on the boundary condition: plus for Neumann and minus for Dirichlet. Now, if we think of \bar{z} and \bar{w} as being in \mathcal{H}^- , correlation functions (2) and (3) are correctly given by those of the holomorphic fields on the entire complex plane,

$$\begin{aligned}
\langle X(z)X(w) \rangle &= \ln(z-w) \\
\langle \psi(z)\psi(w) \rangle &= 1/(z-w)
\end{aligned}$$

To summarize, when considering scattering off p -branes, one can replace $\bar{X}^\mu(\bar{z})$ by $D^\mu_\nu X^\nu(z)$, and analogously for the fermions, where

$$D^\mu_\nu = \begin{bmatrix} 1 & & & & \\ & \ddots & & & \\ & & 1 & & \\ & & & -1 & \\ & & & & \ddots \\ & & & & & -1 \end{bmatrix}.$$

Now the amplitude takes the form

$$\begin{aligned}
A &= \int \frac{dz_1 d\bar{z}_1 dz_2 d\bar{z}_2}{V_{CKG}} \varepsilon_{1\mu\lambda} D^\lambda_\nu \varepsilon_{2\sigma\eta} D^\eta_\kappa \\
&\quad \langle V_{-1}^\nu(Dp_1, \bar{z}_1) V_{-1}^\mu(p_1, z_1) V_0^\sigma(p_2, z_2) V_0^\kappa(Dp_2, \bar{z}_2) \rangle
\end{aligned}$$

Traditionally, the conformal Killing volume in the denominator is cancelled by fixing the positions

of the vertex operators on the world sheet and inserting a Koba-Nielson factor. Readers are referred to [58,59,62] for details. With a convenient choice of the vertex operator positions,

$$\{\bar{z}_1, z_1, z_2, \bar{z}_2\} = \{-iy, iy, i, -i\}$$

the amplitude reduces to

$$A = \int_0^1 dy (1-y^2) \varepsilon_{1\mu\lambda} D^\lambda_\nu \varepsilon_{2\sigma\eta} D^\eta_\kappa \langle V_{-1}^\nu(Dp_1, -iy) V_{-1}^\mu(p_1, iy) V_0^\sigma(p_2, i) V_0^\kappa(Dp_2, -i) \rangle \quad (4)$$

Evaluating the necessary correlation function we find

$$A = \int_0^1 dy \left[\frac{4y}{(1+y)^2} \right]^s \left[\frac{(1-y)^2}{(1+y)^2} \right]^t \times \left[\frac{1}{1-y^2} a_1 - \frac{1-y}{4y(1+y)} a_2 \right]$$

where a_1 and a_2 contain the dependence on the polarizations and momenta, but are independent of y . To do the y -integral, it is convenient to perform a change of variable,

$$y = \frac{1-\sqrt{x}}{1+\sqrt{x}},$$

which brings it to the form

$$A = \int_0^1 dx (1-x)^s x^t (-a_1 x^{-1} + a_2 (1-x)^{-1}).$$

This is a well known integral representation of the Euler Beta function

$$A = \frac{\Gamma(t)\Gamma(s)}{\Gamma(1+s+t)} (sa_1 - ta_2) \quad (5)$$

For completeness, we list the explicit expressions for a_1 and a_2 below:

$$\begin{aligned} a_1 &= \text{Tr}(\varepsilon_1 \cdot D) p_1 \cdot \varepsilon_2 \cdot p_1 - p_1 \cdot \varepsilon_2 \cdot D \cdot \varepsilon_1 \cdot p_2 \\ &\quad - p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D \cdot p_1 - p_1 \cdot \varepsilon_2^T \cdot \varepsilon_1 \cdot D \cdot p_1 \\ &\quad - p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot p_2 + \frac{s}{2} \text{Tr}(\varepsilon_1 \cdot \varepsilon_2^T) \\ &\quad + \{1 \longleftrightarrow 2\} \\ a_2 &= \text{Tr}(\varepsilon_1 \cdot D) (p_1 \cdot \varepsilon_2 \cdot D \cdot p_2 + p_2 \cdot D \cdot \varepsilon_2 \cdot p_1) \end{aligned}$$

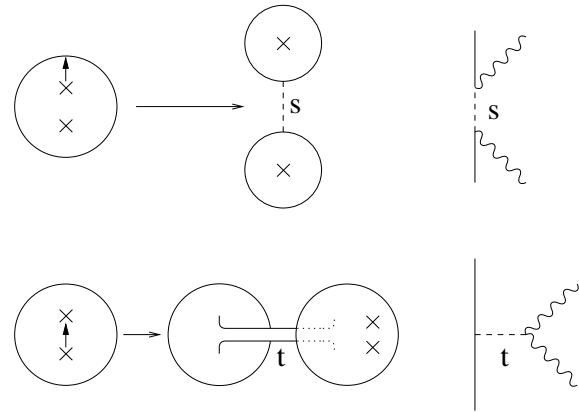


Figure 2. Factorization of the closed string two-point function.

$$\begin{aligned} &+ p_2 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2) + p_1 \cdot D \cdot \varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2 \\ &- p_2 \cdot D \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D \cdot p_1 + \frac{s}{2} \text{Tr}(\varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D) \\ &- \frac{s}{2} \text{Tr}(\varepsilon_1 \cdot \varepsilon_2^T) \\ &- \frac{s+t}{2} \text{Tr}(\varepsilon_1 \cdot D) \text{Tr}(\varepsilon_2 \cdot D) \\ &+ \{1 \longleftrightarrow 2\} . \end{aligned}$$

The amplitude found in (5) exhibits the Regge-pole structure familiar from the Veneziano amplitudes. The poles in the s -channel occur as one of the vertex operators approaches the boundary of the world sheet, while the t -channel poles occur as the vertex operators approach each other. (See figure 2). The fact that the amplitude can be expanded *either* in the s -channel or in the t -channel poles is a characteristic feature of world-sheet duality. Another consequence of this structure is the behavior of the amplitude for large energies at fixed scattering angle. There, one can apply the Stirling's approximation

$$\Gamma(u) = \sqrt{2\pi} u^{u-1/2} e^{-u}$$

and express the scattering amplitude as

$$A = (sa_1 - ta_2) \exp \left(-\frac{\alpha'}{2} s F(\phi) \right)$$

where

$$F(\phi) = \sin^2 \frac{\phi}{2} \log \sin^2 \frac{\phi}{2} + \cos^2 \frac{\phi}{2} \log \cos^2 \frac{\phi}{2}.$$

We have re-instated the factor of α' . The form factor falls exponentially at a rate proportional to α' , implying that the effective thickness of D-branes is of order the string length and increases with the energy. This is to be contrasted with the scattering off a point particle in quantum field theory, where the fixed angle amplitude falls off as a power of s .

Scattering amplitudes with the Regge-pole structure and the exponential fall-off of the form factors is a general feature of all the D-branes except for the D-instanton (sometimes referred to as the -1 brane). The D-instantons are described by world sheets with Dirichlet boundary condition imposed on *all* the coordinates X^M , $0 \leq M \leq 9$. The fact they are special can be understood on kinematical grounds. A D-instanton breaks translation invariance in all directions including the time. Since there are no ‘parallel’ directions, one of the kinematic invariant variables,

$$s = p_{1\parallel}^2 = p_{2\parallel}^2,$$

is meaningless and should be set to zero. In this limit,

$$\frac{\Gamma(s)\Gamma(t)}{\Gamma(1+t+s)} \rightarrow \frac{1}{st},$$

and the total amplitude simplifies to

$$\begin{aligned} A = & -\frac{1}{t} \text{Tr}(\varepsilon_1) p_1 \cdot \varepsilon_2 \cdot p_1 - \frac{1}{t} \text{Tr}(\varepsilon_2) p_2 \cdot \varepsilon_1 \cdot p_2 \\ & + \frac{1}{t} p_1 \cdot (\varepsilon_2 - \varepsilon_2^T) \cdot (\varepsilon_1 - \varepsilon_1^T) p_2 \\ & - \frac{s+t}{s} \text{Tr}(\varepsilon_1) \text{Tr}(\varepsilon_2) \\ & - \frac{1}{2} (\varepsilon_1 - \varepsilon_1^T) \cdot (\varepsilon_2 - \varepsilon_2^T) \end{aligned} \quad (6)$$

The amplitude no longer exhibits Regge-pole structure and resembles a field theory amplitude instead. We will later show that the D-instanton acts as a source for the dilaton and the R-R scalar, but leaves the Einstein metric flat. The dilaton two-point function diverges. Presumably, this divergence is cancelled against the divergence in the

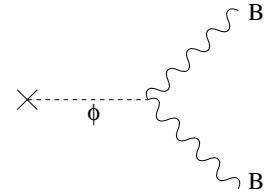


Figure 3. Scattering of the Kalb-Ramond field by a D-instanton.

disconnected diagram where one of the operators is inserted on an annulus, and the other on a disk. The divergence of the disconnected diagram occurs in the limit where the inner boundary of the annulus shrinks to a point, and we expect a cancellation in the spirit of the Fischler-Susskind mechanism. [63]. The anti-symmetric tensor field amplitude is

$$A = \frac{1}{t} p_1 \cdot B_2 \cdot B_1 \cdot p_2 - \frac{1}{2} \text{Tr}(B_1 \cdot B_2).$$

This is precisely how the NS-NS antisymmetric tensor field will scatter off the dilaton background created by the D-instanton, due to the coupling

$$\mathcal{L} = \phi H_{\mu\nu\lambda} H^{\mu\nu\lambda}$$

in the supergravity action (see figure 3). The graviton scattering amplitude vanishes. This is consistent with the fact that there is no tree-level vertex coupling two on-shell gravitons to a dilaton in the supergravity Lagrangian.

2. Elastic scattering of massless R-R states off D-branes

In this section we extend the analysis of closed string scattering to the gauge bosons from the R-R sector [42]. The two-point amplitude is of the general form (1) encountered in the previous section.

In the canonical ghost picture, the vertex operator of an R-R gauge boson with m -form field strength polarization $F_{(m)}$ and momentum k is

given by

$$V(z, \bar{z}) = \mathbb{F}_{(m)}^{\alpha}{}_{\beta} V_{(-1/2)\alpha}(z) \bar{V}_{(-1/2)\beta}^{\beta}(\bar{z}) \quad (\text{IIA})$$

$$V(z, \bar{z}) = \mathbb{F}_{(m)}^{\alpha\beta} V_{(-1/2)\alpha}(z) \bar{V}_{(-1/2)\beta}(\bar{z}) \quad (\text{IIB})$$

with

$$\begin{aligned} V_{(-1/2)\alpha}(z) &= e^{-\phi(z)/2} S_{\alpha}(z) e^{ikX(z)} \\ V_{(-1/2)}^{\beta}(z) &= e^{-\phi(z)/2} S^{\beta}(z) e^{ikX(z)} \end{aligned}$$

and

$$\mathbb{F}_{(m)} = \frac{1}{m!} F_{\mu_1 \dots \mu_m} \gamma^{\mu_1} \dots \gamma^{\mu_m}.$$

We have distinguished between the type IIA theory where the allowed values of m are 2 and 4, and the type IIB theory where $m = 1, 3, 5$. Note that the two-point function of these vertex operators on a disk has precisely the right amount of total superghost charge.

We will use a representation in which the 32×32 Dirac gamma matrices are off-diagonal:

$$\gamma^{\mu} = \begin{pmatrix} 0 & \gamma^{\mu\alpha\beta} \\ \gamma_{\alpha\beta}^{\mu} & 0 \end{pmatrix}.$$

We normalize the γ^{μ} so that $\{\gamma^{\mu}, \gamma^{\nu}\} = -2\eta^{\mu\nu}$, and we pick our representation so that

$$\gamma_{11} = \gamma^0 \dots \gamma^9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The main new ingredient in the discussion of the R-R sector are the spin fields, which assume a simple form upon bosonization of the world-sheet fermions,

$$\begin{aligned} S^{\alpha}(z) &= C^{\alpha} e^{i\lambda_i^{\alpha}\phi_i(z)} \\ \frac{i}{\sqrt{2}} (\psi^{2i-1}(z) \pm i\psi^{2i}(z)) &= e^{\pm i\phi_i(z)} \end{aligned}$$

where $i = 1 \dots 5$, and λ^{α} 's are the weight vectors in the chiral or anti-chiral conjugacy class of $SO(1, 9)$. The cocycle operators, C^{α} , impose the correct anticommutation relations. The Dirichlet or Neumann boundary conditions on ψ translate into the following boundary conditions on the spin fields,

$$\begin{aligned} \bar{S}^{\alpha}(\bar{z}) &= M^{\alpha\beta} S_{\beta}(z) \quad (\text{type IIA}) \\ \bar{S}_{\alpha}(\bar{z}) &= M_{\alpha}{}^{\beta} S_{\beta}(z) \quad (\text{type IIB}) \end{aligned}$$

where

$$M = \gamma^0 \gamma^1 \dots \gamma^p.$$

The appearance of such a boundary condition was almost inevitable given the $SO(1, p) \times SO(9-p)$ symmetry of the theory. Just as we did in the previous section, we can extend the definition of spin fields from \mathcal{H}^+ to the entire complex plane by defining

$$\begin{aligned} S_{\alpha}(z) &= \begin{cases} S_{\alpha}(z) & z \in \mathcal{H}^+ \\ M_{\alpha\beta} \bar{S}^{\beta}(z) & z \in \mathcal{H}^- \end{cases} \quad (\text{type IIA}) \\ S_{\alpha}(z) &= \begin{cases} S_{\alpha}(z) & z \in \mathcal{H}^+ \\ M_{\alpha}{}^{\beta} \bar{S}_{\beta}(z) & z \in \mathcal{H}^- \end{cases} \quad (\text{type IIB}) \end{aligned}$$

In terms of the world sheet variables extended to the entire complex plane, the amplitude becomes

$$\begin{aligned} A &= \int \frac{dz_1 d\bar{z}_1 dz_2 d\bar{z}_2}{V_{CKG}} (M \mathbb{F}^{(1)})^{\alpha\beta} (M \mathbb{F}^{(2)})^{\gamma\delta} \\ &\quad \langle V_{(-1/2)\alpha}(p_1, z_1) V_{(-1/2)\beta}(p_1, \bar{z}_1) \\ &\quad \quad V_{(-1/2)\gamma}(p_2, z_2) V_{(-1/2)\delta}(p_2, \bar{z}_2) \rangle \end{aligned}$$

The four-point amplitude of the spin fields, which is needed here, was calculated in [58,64],

$$\begin{aligned} \langle S_{\alpha}(z_1) S_{\beta}(z_2) S_{\gamma}(z_3) S_{\delta}(z_4) \rangle &= \\ \frac{z_{14} z_{23} \gamma_{\alpha\beta}^{\mu} \gamma_{\gamma\delta}^{\mu} - z_{12} z_{34} \gamma_{\alpha\delta}^{\mu} \gamma_{\mu\beta\gamma}^{\mu}}{2(z_{12} z_{13} z_{14} z_{23} z_{24} z_{34})^{3/4}}. \end{aligned} \quad (7)$$

The rest of the calculation follows exactly the same steps as in the previous section, and we find that the amplitude has an analogous form,

$$A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} [(s+t)P_1 + sP_2] \quad (8)$$

where P_1 and P_2 arise from two terms in (7),

$$\begin{aligned} P_1 &= \text{Tr} \left(P \mathbb{F}_{(m)}^{(1)} M \gamma^{\mu} \right) \text{Tr} \left(P \mathbb{F}_{(n)}^{(2)} M \gamma_{\mu} \right) \\ P_2 &= \text{Tr} \left(P \mathbb{F}_{(m)}^{(1)} M \gamma^{\mu} \mathbb{F}_{(n)}^{(2)} M \gamma_{\mu} \right). \end{aligned}$$

and P is the chiral projection operator

$$P = \frac{1 + \gamma_{11}}{2}$$

The final expression (8) for the scattering of a massless R-R boson has the same Regge-pole structure as the amplitude of massless NS-NS bosons, (5). Since the R-R bosons couple to the

R-R charge, we conclude that the effective size of the D-brane R-R charge distribution is of the order $\sqrt{\alpha'}$, when measured by the scattering of massless closed strings.

3. Scattering off black p -branes in supergravity

In this section we compare the scattering off D-branes with the scattering off p -brane solitons of low-energy supergravity. The fact that two amplitudes agree for large impact parameters provides a dynamical evidence that the D-branes are indeed the string theoretic realizations of the supergravity p -branes.

Let us begin by reviewing the basic properties of the extreme R-R charged p -brane solutions of the supergravity equations of motion. General solutions of this type were first written down in [13]:

$$\begin{aligned} ds^2 &= A^{-1/2} (-dt^2 + dx_1^2 + \dots + dx_p^2) \\ &\quad + A^{1/2} (dy^2 + y^2 d\Omega_{8-p}^2) \\ e^{-2\phi} &= A^{(p-3)/2} \\ F_{(p+2)} &= \frac{Q}{y^{8-p}} A^{-2} dt \wedge dx_1 \wedge \dots \wedge dx_p \wedge dy \end{aligned} \quad (9)$$

where $F_{(p+2)}$ is the R-R field strength coupling to the brane, and

$$A = 1 + \frac{2}{7-p} \frac{Q}{y^{7-p}}. \quad (10)$$

While [13] discussed only $0 \leq p \leq 6$, solutions (9) and (10) may be extrapolated in an obvious way to $p = 7$ [65] and $p = 8$. For the 7-brane the solution is (9) with

$$A = 1 + 2Q \ln(y/y_0),$$

while for the 8-brane,

$$A = 1 + 2Qy = 1 + 2Q|x_9|.$$

A new feature we find for $p = 7$ and 8 is that A grows with the distance from the p -brane. Thus, the geometry is not asymptotically flat.

The D-instanton ($p = -1$) is a special case because it requires a Euclidean continuation [42,65].

The D-instanton is a source of the R-R scalar, which is essentially a ten-dimensional axion. In order to make its axionic properties manifest, we are going to use its dual, 8-form, description. Our goal, therefore, is to find an instanton solution to the following euclidean action,

$$S = \int d^{10}x \sqrt{G} \left[e^{-2\phi} (-R + 4(\partial\phi)^2) + \frac{1}{2 \cdot 9!} F_{(9)}^2 \right]$$

It is not hard to verify that the following is a solution:

$$\begin{aligned} ds^2 &= A^{1/2} (dy^2 + y^2 d\Omega_9^2) \\ e^{-2\phi} &= A^{-2} \\ F_{(9)} &= \frac{Q}{y^9} A^{-2} * dy \end{aligned}$$

where

$$A = 1 + \frac{1}{4} \frac{Q}{y^8}.$$

This solution has the following interesting feature: the Einstein metric,

$$g^{\mu\nu} = G^{\mu\nu} e^{-\phi/2} = \delta_{\mu\nu},$$

is flat! Since the Einstein metric describes the physical gravitational field, we conclude that the R-R charged instanton in 10 dimensions emits a dilaton, but no gravitational field. This explains the vanishing of the graviton scattering amplitude off of a D-instanton.

In order to compute the scattering at large impact parameter, we expand the background in powers of

$$\begin{aligned} \lambda &= \frac{1}{7-p} \frac{Q}{y^{7-p}} \\ &= \frac{2\pi^{(9-p)/2}}{\Gamma((9-p)/2)} \int \frac{d^{9-p}q}{(2\pi)^{9-p}} e^{iq \cdot y} \frac{Q}{q^2}. \end{aligned}$$

To first order in λ , the string metric is given by

$$G^{\mu\nu} = \eta^{\mu\nu} - \lambda D^{\mu\nu}$$

The large impact parameter limit of the amplitude for strings to scatter off a soliton background is obtained by expanding the relevant part of the action,

$$S = \int d^{10}x \sqrt{G} \frac{1}{2 \cdot n!} F_{(n)}^2$$

to leading order in λ . To this order, we find

$$\begin{aligned} \delta S = & \lambda \left\{ (\text{Tr}D) F_{\mu_1 \dots \mu_n}^{(1)} F^{(2) \mu_1 \dots \mu_n} \right. \\ & \left. + n D_{\mu_1 \nu_1} \eta_{\mu_2 \nu_2} \dots \eta_{\mu_n \nu_n} F^{(1) \mu_1 \dots \mu_n} F^{(2) \nu_1 \dots \nu_n} \right\} \end{aligned} \quad (11)$$

To make explicit comparisons with string theory we expand this expression in terms of $SO(1, p) \times SO(9 - p)$ invariants,

$$\begin{aligned} \delta S \sim & \frac{1}{t} \sum_{a+b=n} c^{(a,b)} F_{m_1 \dots m_a M_1 \dots M_b}^{(1)} F^{(2) m_1 \dots m_a M_1 \dots M_b} \end{aligned}$$

After some combinatorics, one finds from (11) that

$$c_{\text{field}}^{(a,b)} = (4 - p + a - b) \binom{n}{a} \quad (12)$$

In order to compare this with the large impact parameter limit of the string theory amplitude, we examine the leading t -channel pole in equation (8),

$$A = \frac{1}{t} (P_1 + P_2).$$

In order to express this in terms of $SO(1, p) \times SO(9 - p)$ invariants as we did for the field theory amplitudes, we must evaluate the trace of the γ -matrices. The main formula one uses is the general (anti)-commutator of anti-symmetrized gamma matrices:

$$\begin{aligned} & \left[\gamma^{[\mu_1} \dots \gamma^{\mu_m]}, \gamma_{[\nu_1} \dots \gamma_{\nu_n]} \right]_{(-1)^{mn+1}} = \\ & \sum_{j=1}^m (-1)^{1+mj+j(j+1)/2} \binom{m}{j} \binom{n}{j} \\ & 2^j j! \delta_{[\nu_1}^{[\mu_1} \dots \delta_{\nu_j}^{\mu_j} \gamma^{\mu_{j+1}} \dots \gamma^{\mu_m]} \gamma_{\nu_{j+1}} \dots \gamma_{\nu_n]}. \end{aligned}$$

Using this identity, one can show that

$$\begin{aligned} P_1 + P_2 = & -32 c_{\text{string}}^{(a,b)} F_{m_1 \dots m_a M_1 \dots M_b}^{(1)} F^{(2) m_1 \dots m_a M_1 \dots M_b} \end{aligned}$$

with

$$\begin{aligned} c_{\text{string}}^{(a,b)} = & 8 \left((p+2) \delta_{n-p-2} \delta_{b-1} + \delta_{n-p} \delta_b \right. \\ & - \delta_{n+p-8} \delta_a - (10-p) \delta_{n+p-10} \delta_{a-1} \\ & \left. - (-1)^{np+p(p+1)/2+n(n+1)/2+a} \right. \\ & \left. (4-p+a-b) \binom{n}{a} \right). \end{aligned}$$

It is a somewhat non-trivial fact that $c_{\text{field}}^{(a,b)}$ and $c_{\text{string}}^{(a,b)}$ are identical! Altogether there are 25 different scattering processes where our field theory analysis serves as a check on the string theory computation: $n = 1, 3, 5$ for p odd and $n = 2, 4$ for p even, with $-1 \leq p \leq 8$. We have checked the agreement between string theory and field theory in all of the 25 possible cases. The consistent agreement in the large impact parameter scattering is a strong piece of evidence for regarding the extreme black p -branes of supergravity as the low-energy descriptions of the Dirichlet p -branes.

4. Type I four-point amplitudes and D-branes

An astute reader may have noticed that our closed string two-point scattering amplitudes closely resemble the type I open string four-point amplitudes. This relation was made precise in a very nice paper by Garousi and Myers [43]. Upon appropriately fixing the residual conformal Killing volume, they found that the two amplitudes are *identical* upon a certain identification between the momenta and polarizations.

Consider a vertex operator for the vector boson of type I theory with momentum k and polarization ζ , which is to be integrated over the real axis.

$$V_0 = \zeta_\mu \left(\frac{1}{2} \partial_t X^\mu + i 2k \cdot \psi \psi^\mu \right) e^{ik \cdot X}(\sigma) \quad (13)$$

Using the world sheet doubling trick, we may write the operators in the -1 and 0 picture in terms of the holomorphic only,

$$\begin{aligned} V_{-1}^\mu(z, 2k) &= e^{-\phi} \psi^\mu e^{i 2k \cdot X}(z) \\ V_0^\mu(z, 2k) &= (\partial X^\mu + i 2k \cdot \psi \psi^\mu) e^{i 2k \cdot X}(z) \end{aligned}$$

Let us consider the $2 \rightarrow 2$ scattering of type I open strings with momentum k_i and polarizations ζ_i for $i = 1 \dots 4$. The amplitude is written explicitly as

$$\begin{aligned} & A(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) \\ & = \int \frac{dx_1 dx_2 dx_3 dx_4}{V_{CKG}} \\ & \quad \langle \zeta_1 \cdot V_0(2k_1, x_1) \zeta_2 \cdot V_0(2k_2, x_2) \\ & \quad \zeta_3 \cdot V_{-1}(2k_3, x_3) \zeta_4 \cdot V_{-1}(2k_4, x_4) \rangle \end{aligned}$$

For future use, it is convenient to introduce the Mandelstam variables

$$\begin{aligned} s &= 4k_1 \cdot k_2 = 4k_3 \cdot k_4, \\ t &= 4k_1 \cdot k_4 = 4k_2 \cdot k_3, \\ u &= 4k_1 \cdot k_3 = 4k_2 \cdot k_4. \end{aligned}$$

The traditional method for cancelling the conformal Killing volume is to set the vertex operators at

$$\{x_1, x_2, x_3, x_4\} = \{0, x, 1, \infty\}$$

and to integrate x from 0 to 1. An alternative is to fix the vertex operators at

$$\{x_1, x_2, x_3, x_4\} = \{-x, x, 1, -1\}$$

The four-point amplitude then takes the form

$$\begin{aligned} A = \int_0^1 dx (1-x^2) \zeta_{1\mu} \zeta_{2\nu} \zeta_{3\sigma} \zeta_{4\kappa} \\ \langle V_{-1}^\mu(k_1, -x) V_{-1}^\nu(k_2, x) V_0^\sigma(k_3, 1) V_0^\kappa(k_4, -1) \rangle \end{aligned} \quad (14)$$

This is identical to the integral encountered in (4) under the identification

$$\begin{aligned} 2k_1^\mu &\rightarrow (D \cdot p_1)^\mu & 2k_2^\mu &\rightarrow p_1^\mu \\ 2k_3^\mu &\rightarrow p_2^\mu & 2k_4^\mu &\rightarrow (D \cdot p_2)^\mu \\ \zeta_{2\mu} \otimes \zeta_{1\nu} &\rightarrow \varepsilon_{1\mu\lambda} D_\nu^\lambda \\ \zeta_{3\mu} \otimes \zeta_{4\nu} &\rightarrow \varepsilon_{2\mu\lambda} D_\nu^\lambda. \end{aligned} \quad (15)$$

The result of evaluating (4) was

$$A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} (sa_1 - ta_2)$$

On the other hand, the type I four-point function has the well known form [66]

$$A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} K(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) \quad (16)$$

where $K(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4)$ is the standard kinematic factor,

$$\begin{aligned} K(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) = & \\ -16k_2 \cdot k_3 k_2 \cdot k_4 \zeta_1 \cdot \zeta_2 \zeta_3 \cdot \zeta_4 & \\ -4k_1 \cdot k_2 (\zeta_1 \cdot k_4 \zeta_3 \cdot k_2 \zeta_2 \cdot \zeta_4 + \zeta_2 \cdot k_3 \zeta_4 \cdot k_1 \zeta_1 \cdot \zeta_3) & \\ + \zeta_1 \cdot k_3 \zeta_4 \cdot k_2 \zeta_2 \cdot \zeta_3 + \zeta_2 \cdot k_4 \zeta_3 \cdot k_1 \zeta_1 \cdot \zeta_4 & \\ + \left\{ 1, 2, 3, 4 \rightarrow 1, 3, 2, 4 \right\} & \\ + \left\{ 1, 2, 3, 4 \rightarrow 1, 4, 3, 2 \right\}. & \end{aligned} \quad (17)$$

The fact that the amplitude for scattering off a D-brane (4) and the open string four point amplitude (14) take on identical forms allows us to identify the kinematic factors,

$$(sa_1 - ta_2) = K(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) \quad (18)$$

One can verify that a_1 and a_2 found in section 1 indeed satisfy (18) under the identification (15).

Now, the open string four-point amplitudes have been computed for all combinations of NS and Ramond external states. The amplitudes have the general structure (16) while the kinematic factors are given by [66]:

$$\begin{aligned} K_2(u_1, u_2, u_3, u_4) = & -2k_1 \cdot k_2 \bar{u}_2 \gamma^\mu u_3 \bar{u}_1 \gamma_\mu u_4 \\ + 2k_1 \cdot k_4 \bar{u}_1 \gamma^\mu u_2 \bar{u}_4 \gamma_\mu u_3 \end{aligned} \quad (19)$$

$$\begin{aligned} K_3(u_1, \zeta_2, \zeta_3, u_4) = & 2i\sqrt{2}k_1 \cdot k_4 \bar{u}_1 \gamma \cdot \zeta_2 \gamma \cdot (k_3 + k_4) \gamma \cdot \zeta_3 u_4 \\ - 4i\sqrt{2}k_1 \cdot k_2 (\bar{u}_1 \gamma \cdot \zeta_3 u_4 k_3 \cdot \zeta_2 & \\ - \bar{u}_1 \gamma \cdot \zeta_2 u_4 k_2 \cdot \zeta_3 - \bar{u}_1 \gamma \cdot k_3 u_4 \zeta_2 \cdot \zeta_3) \end{aligned} \quad (20)$$

$$\begin{aligned} K_4(u_1, \zeta_2, u_3, \zeta_4) = & -2i\sqrt{2}k_1 \cdot k_4 \bar{u}_1 \gamma \cdot \zeta_2 \gamma \cdot (k_3 + k_4) \gamma \cdot \zeta_4 u_3 \\ - 2i\sqrt{2}k_1 \cdot k_2 \bar{u}_1 \gamma \cdot \zeta_4 \gamma \cdot (k_2 + k_3) \gamma \cdot \zeta_2 u_3 . \end{aligned} \quad (21)$$

The observation that the two point amplitude of a closed string in a background of a D-brane is identical to the four point function of type I theory is extremely useful because one can apply this identification to compute scattering amplitudes of all types of closed strings: NS-NS, R-R, NS-R and R-NS. For the sake of illustration, consider a process where a massless NS-NS boson with momentum p_1 and polarization $\varepsilon_{\mu\nu}$ is absorbed and a R-R boson with momentum p_2 and polarization F is emitted by the D-brane. The amplitude for such a process is given by

$$A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} K_3(u_1, \zeta_2, \zeta_3, u_4)$$

where we identify

$$\begin{aligned} 2k_1^\mu &\rightarrow p_1^\mu & 2k_2^\mu &\rightarrow p_2^\mu \\ 2k_3^\mu &\rightarrow (D \cdot p_2)^\mu & 2k_4^\mu &\rightarrow (D \cdot p_1)^\mu \\ u_{1A} \otimes u_{4B} &\rightarrow (P \not{F} M)_{AB} \\ \zeta_{2\mu} \otimes \zeta_{3\nu} &\rightarrow \varepsilon_{2\mu\lambda} D_\nu^\lambda \end{aligned}$$

Using similar identifications, Garousi and Meyers were able to derive all $1 \rightarrow 1$ amplitudes for closed strings.

Let us now consider the process illustrated in figure 1.B. This is the four-point function of open strings with ends attached to the D-brane. The vertex operator for such an open string is obtained by T-dualizing the vertex operator (13) along the directions transverse to the brane,

$$V_0^M = \left(\frac{1}{2} \partial_n X^M + i 2k \cdot \psi \psi^M \right) e^{ik \cdot X}(\sigma).$$

Now, since the boundary conditions along these directions are Dirichlet,

$$\begin{aligned} X^M(z) &= -\bar{X}^M(\bar{z}) \\ \psi^M(z) &= -\bar{\psi}^M(\bar{z}) \end{aligned} \quad (z, \bar{z}) \in \partial \mathcal{H}^+$$

the vertex operator, when expressed in strictly holomorphic variables, becomes identical to what we found in the case of Neumann boundary condition (with an extra constraint that the momentum k be parallel to the brane.)

$$\begin{aligned} V_{-1}^M(z, 2k) &= e^{-\phi} \psi^M e^{i 2k \cdot X}(z) \\ V_0^\mu(z, 2k) &= (\partial X^M + i 2k \cdot \psi \psi^M) e^{i 2k \cdot X}(z) \end{aligned}$$

Thus, the amplitude of figure 1.B is identical to the open string four-point function,

$$A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(1+s+t)} K$$

with the restriction that all the momenta are parallel to the brane. This is hardly a surprise given the fact that the low-energy effective dynamics on a Dirichlet p -brane is described by a dimensional reduction of $N = 1$ supersymmetric Yang-Mills theory from 10-dimensions to $p + 1$ dimensions.

5. Absorption and Hawking emission by D-branes

Finally, let us consider the process illustrated in figure 1.C, that is, the absorption of a massless closed string state leading to a pair creation of open strings attached to the D-brane [44]. To begin, let us focus on the NS sector and assign momenta p_1 and p_2 to the open string states and momentum q to the closed string. The open string

momenta p_1 and p_2 are restricted to lie within the D-brane world volume. The D-brane kinematics is such that only the longitudinal momentum is conserved,

$$p_1 + p_2 + q_{\parallel} = 0. \quad (22)$$

Ordinarily in massless three-point amplitudes conservation of momentum constrains the kinematics completely but, in the presence of D-branes, the non-conservation of momentum in the directions transverse to the brane leaves some freedom. Here, from the conservation of the longitudinal momentum, it follows that there is exactly one kinematic invariant in this problem, which we call t ,

$$t = 2p_1 \cdot q = 2p_2 \cdot q = -2p_1 \cdot p_2. \quad (23)$$

The leading order contribution to this amplitude is evaluated on a disk with two operators on the boundary and one in the bulk. We proceed by mapping the disk to the upper half-plane,

$$A = \int \frac{dz_1 dz_2 d^2 z_3}{V_{CKG}} \langle V_1(z_1) V_2(z_2) V_3(z_3, \bar{z}_3) \rangle \quad (24)$$

where z_1 and z_2 are integrated only along the real axis. The vertex operators are the same as the ones used in the preceding sections and, after the doubling of the world sheet, the amplitude becomes

$$\begin{aligned} A &= \int \frac{dz_1 dz_2 dz_3 d\bar{z}_3}{V_{CKG}} \xi_\mu^1 \xi_\nu^2 \epsilon_{\sigma\lambda} D^\lambda \eta \\ &\quad \langle V_0^\mu(z_1, 2p_1) V_0^\nu(z_2, 2p_2) V_{-1}^\sigma(z_3, q) V_{-1}^\eta(\bar{z}_3, D \cdot q) \rangle \end{aligned} \quad (25)$$

This is precisely the form of the correlation function encountered previously in the computation of the open string 4-point function (14), if we identify

$$\begin{aligned} 2k_1 &\rightarrow 2p_1 & 2k_2 &\rightarrow 2p_2 \\ 2k_3 &\rightarrow q & 2k_4 &\rightarrow D \cdot q \\ \zeta_1 &\rightarrow \xi_1 & \zeta_2 &\rightarrow \xi_2 \\ \zeta_3 \otimes \zeta_4 &\rightarrow \varepsilon \cdot D \end{aligned} \quad (26)$$

This allows us to use a short-cut similar to that found in [43] for the closed string two-point function. We fix the vertex operators at $\{z_1, z_2, z_3, \bar{z}_3\} = \{-x, x, i, -i\}$, which corresponds to placing the closed string vertex at $z = i$

and constraining the open string vertex operators to lie symmetrically on the real axis. Notice that this is related to the calculations in section 1 by a change of variables, $y = -ix$. Recalling that there is only one kinematic invariant in this problem, we set $s = 4k_1k_2 = -2t$. After these replacements the amplitude takes the form:

$$A = \int_{-\infty}^{\infty} dx \left[\frac{(1+x^2)^2}{16x^2} \right]^t \frac{1}{1+x^2} \left(a_1 + \frac{a_2}{2} \right) \quad (27)$$

Performing the integral we find

$$A = (-2ta_1 - ta_2) \frac{\Gamma(-2t)}{\Gamma(1-t)^2} \quad (28)$$

It is clear that $(-2ta_1 - ta_2)$ is the same kinematic factor $K = (sa_1 - ta_2)$ that we encountered above, with $s = -2t$. Thus,

$$A = \frac{\Gamma(-2t)}{\Gamma(1-t)^2} K(1, 2, 3) \quad (29)$$

where the kinematic factor is obtained from that of type I theory, (17), by the identifications (26). This is the main result of this section.

The explicit formula for the Neveu-Schwarz amplitudes is, therefore,

$$\begin{aligned} K(1, 2, 3) = & [t(-q \cdot \xi_2 \xi_1 \cdot D \cdot q g^{\mu\nu} - q \cdot \xi_1 \xi_2 \cdot D \cdot q g^{\mu\nu} \\ & - 4\xi_1 \cdot \xi_2 p_1^\nu p_2^\mu - 4\xi_1 \cdot \xi_2 p_1^\mu p_2^\nu \\ & - 2p_1 \cdot \xi_2 q^\nu \xi_1^\mu + 4q \cdot \xi_2 p_1^\nu \xi_1^\mu \\ & - 2p_1 \cdot \xi_2 (D \cdot q)^\mu \xi_1^\nu + 4\xi_2 \cdot D \cdot q p_1^\mu \xi_1^\nu \\ & - 2p_2 \cdot \xi_1 q^\nu \xi_2^\mu + 4q \cdot \xi_1 p_2^\nu \xi_2^\mu \\ & - 2p_2 \cdot \xi_1 (D \cdot q)^\mu \xi_2^\nu + 4\xi_1 \cdot D \cdot q p_2^\mu \xi_2^\nu) \\ & + t^2(-\xi_1 \cdot \xi_2 g^{\mu\nu} + 2\xi_1^\nu \xi_2^\mu + 2\xi_1^\mu \xi_2^\nu)] \\ & (\varepsilon \cdot D)_{\mu\nu} \end{aligned} \quad (30)$$

The gauge invariance follows automatically from that of the type I kinematic factor. When we take the polarization of the closed string to be traceless and strictly transverse to the D-brane, the kinematic factor simplifies considerably, and one is left with

$$A \sim \frac{\Gamma(-2t)}{\Gamma(1-t)^2} t^2 (\xi^1 \cdot \varepsilon \cdot \xi^2 + \xi^2 \cdot \varepsilon \cdot \xi^1) \quad (31)$$

In the low t approximation, and for symmetric ε (i. e. those describing gravitons), this amplitude

reduces to

$$A = t \varepsilon_{MN} \xi_1^M \xi_2^N$$

which coincides with the three point amplitude obtained from following term in the effective action

$$\mathcal{L} = \int d^{p+1}x \partial_m \phi^M \partial^m \phi^N G_{MN}$$

The fields $\phi^M(x^m)$ specify the transverse location of the D-brane. This is the leading term in the static gauge expansion of the Nambu-Goto action.

The amplitudes for states involving the R sector are again of the general form found in (29), with the kinematic factors obtained from the type I ones, (19-21), under appropriate identification of kinematic variables. Let us illustrate this with some examples.

First, consider pair creation of NS open strings by an R-R closed string with n -form polarization $F_{\mu_1\mu_2\dots\mu_n}$. In this case, the correlation function is of the form

$$\begin{aligned} \xi_\mu \xi_\nu (P \mathcal{F} M)^{AB} & \langle V_0^\mu(z_1, 2p_1) V_{-1}^\nu(z_2, 2p_2) \\ & V_{-1/2A}(z_3, q) V_{-1/2B}(\bar{z}_3, D \cdot q) \rangle \end{aligned}$$

where just as in section 2,

$$\begin{aligned} \mathcal{F} &= \frac{1}{n!} F_{\mu_1\mu_2\dots\mu_n} \gamma^{\mu_1} \gamma^{\mu_2} \dots \gamma^{\mu_n}, \\ M &= \gamma^0 \gamma^1 \dots \gamma^p, \\ P &= (1 + \gamma_{11})/2. \end{aligned}$$

The amplitude reduces to

$$A = \frac{\Gamma(-2t)}{\Gamma(1-t)^2} K(1, 2, 3), \quad (32)$$

with the kinematic factor obtained from that in type I theory, (20), by the following substitutions,

$$\begin{aligned} 2k_1 &\rightarrow D \cdot q & 2k_2 &\rightarrow 2p_1 \\ 2k_3 &\rightarrow 2p_2 & 2k_4 &\rightarrow q \\ \zeta_2 &\rightarrow \xi_1 & \zeta_3 &\rightarrow \xi_2 \\ u_4 \otimes u_1 &\rightarrow P \mathcal{F} M \end{aligned} .$$

This gives

$$\begin{aligned} K(1, 2, 3) = t \text{Tr} [P \mathcal{F} M & (\gamma \cdot \xi_1 \gamma \cdot (p_2 + q/2) \gamma \cdot \xi_2 + 4(p_2 \cdot \xi_1) \gamma \cdot \xi_2 \\ & - 4(p_1 \cdot \xi_2) \gamma \cdot \xi_1 - 4(\xi_1 \cdot \xi_2) \gamma \cdot p_2)] \end{aligned}$$

Similarly, consider exciting two open string fermions with polarizations v_1^A and v_2^B by an incident massless NS-NS boson with polarization $\varepsilon_{\mu\nu}$. Now, the correlation function takes the form

$$v_1^A v_2^B (\varepsilon \cdot D)_{\mu\nu} \\ \langle V_{-1/2A}(z_1, 2p_1) V_{-1/2B}(z_2, 2p_2) \\ V_0^\mu(z_3, q) V_{-1}^\nu(\bar{z}_3, D \cdot q) \rangle$$

The relevant kinematic factor is obtained from (20) by the following substitutions,

$$\begin{aligned} 2k_1 &\rightarrow 2p_2 & 2k_2 &\rightarrow q \\ 2k_3 &\rightarrow D \cdot q & 2k_4 &\rightarrow 2p_1 \\ u_4 &\rightarrow v_1 & u_1 &\rightarrow v_2 \\ \zeta_2 \otimes \zeta_3 &\rightarrow (\varepsilon \cdot D) \end{aligned}$$

Thus, the amplitude for two open string fermions to produce a NS-NS closed string state is again of the general form (32) with the kinematic factor

$$K(1, 2, 3) = t (\varepsilon \cdot D)_{\mu\nu} \\ [\bar{v}_2 \gamma^\mu \gamma \cdot (D \cdot q + 2p_1) \gamma^\nu v_1 + 4 \bar{v}_2 \gamma^\nu v_1 (D \cdot q)^\mu \\ - 4 \bar{v}_2 \gamma^\mu v_1 q^\nu - 4 \bar{v}_2 (\gamma \cdot D \cdot q) v_1 g^{\mu\nu}]$$

It is straightforward to extend this program to other combinations of Neveu-Schwarz and Ramond vertex operators.

It is interesting that these amplitudes occupy an intermediate position between the conventional three-point and four-point amplitudes as far as the number of kinematic invariants goes. This is a consequence of the ‘partial conservation of momentum’ unique to D-branes. These amplitudes have other interesting new features. They decay exponentially for large t , which indicates the softness of strings at high energies, and have a sequence of poles at half-odd-integer values of t . What is surprising is that they also contain a sequence of zeros for integer values of t . The special role played by these values of t is related to the massive open string states appearing in the operator product expansion of the open string vertex operators when they collide on the world sheet (see figure (4)). These states have masses

$$m^2 = n/\alpha' = n/2 \quad (33)$$

for integer n , and indeed

$$t = -2p_1 \cdot p_2 = -(p_1 + p_2)^2 = m^2 = n/2. \quad (34)$$

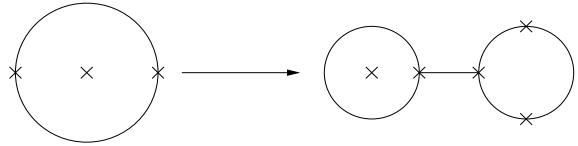


Figure 4. Factorization of the world sheet giving rise to poles in t .

Interestingly, for even n these amplitudes have zeros instead of poles, indicating that these states do not propagate in the internal line of figure 1. Thus, due to the special kinematics of this process, we find an interesting Z_2 selection rule. This interplay between the zeros and the poles is intimately connected with the exponential decay of the amplitude at large t .

6. Conclusions

These lecture notes summarize some of the simplest perturbative calculations that describe interactions of D-branes and elementary strings. The results reveal a consistent physical picture. Since the D-branes are described by open strings whose ends are attached to specified hyperplanes, and couple to closed strings via the conventional open-closed string interactions, they acquire the softness characteristic of strings. Their effective thickness, observed with massless closed string probes, is of order the string scale, $\sqrt{\alpha'}$, and grows with the energy.

One could wonder, however, whether there exists a smaller structure underneath the ‘string halo’ that we have described. A number of interesting recent works have reached the conclusion that the answer is yes, and that this structure becomes apparent only when the D-branes are probed with other D-branes whose velocity is very small [49,50,52–56]. This effect seems to crucially depend on the cancellation of forces at rest between certain types of D-branes, i.e., on the fact that we are studying slow motion in a BPS saturated system. The scattering of D-branes from other D-branes is a complicated problem because, unlike the questions considered here, it is entirely

non-perturbative. Its study in the context of low energy field theory reveals a non-perturbative length scale, $g^{1/3}\sqrt{\alpha'}$, which is completely natural since it is the Planck scale of the 11-dimensional M-theory. We feel, however, that there is much that remains to be learned about the structure of D-branes. We need a clear understanding of which scales govern various processes: at what stage, for instance, do the shorter scales found in slow motion get smeared by the string scale. We should also keep in mind that, if the string coupling g turns out to be of order one, then the string scale and the 11-dimensional Planck scale turn out to be of the same order and should be regarded as competing effects. To conclude, the D-branes have provided us with a remarkable new viewpoint on non-perturbative string theory. No doubt, much remains to be learned from this remarkable tool.

Acknowledgements

We are grateful to S. Gubser, J. Maldacena and L. Thorlacius for collaborations on the material presented here. This work was supported in part by the DOE grant DE-FG02-91ER40671, the NSF Presidential Young Investigator Award PHY-9157482, and the James S. McDonnell Foundation grant No. 91-48.

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